# A mathematical approach to cognitive processes

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*Summary.* A mathematical formalization of cognitive processes based on concepts previously given in Experientia[1] is presented. Cognitive processes are described as a set of exponentially decaying interaction probabilities determined by previous interactions between different elements and an association parameter, the inverse of the cognitive stability. The interactions occur in a 'cognitive string' surrounded by a 'cognitive plasma' which carries the contextual information. The 'complexity' of a cognitive process is proportional to time.
*Key words.* Cognitive processes; elementary reductionism; cognitive stability; dynamics of interaction; hierarchical elements; cognitive string; cognitive plasma; complementarity; complexity.

## *Introduction*

In a previous paper[1], a mechanism for the self-organization of cognitive processes was proposed, according to which different elements composed of sub-elements interact and thereby build up more complicated conceptual structures. As far as the interactions between the elements are mediated by the dynamics of the sub-elements which distribute beyond the volume of one element, and informatory interactions are included in the definition, cognitive self-organization seems to be of general occurrence in nature and can be found in several real systems, such as atoms, molecules, biological cells, organisms, societies, etc.[1]. The concept of shuttling sub-elements is also to be found in games, or in economics in the circulation of money, where the transfer of the sub-elements mediates interactions between the higher elements and contributes to a certain dynamic organization of elements. The evolution of science procedes when *different* experiments and theories are put together in new patterns in which an exclusion principle of informatory elements operates, similarly to the Pauli exclusion principle of electron shells or atomic nuclei. In principle, no new concept is arrived at by repeating an identical experiment or by saying the same thing over and over again. Also in simple cognitive processes, like the formation of words from letters, similar mechanisms operate. In view of the possibility that these phenomena may be contained within a common formalism, an attempt will be made to develop a mathematical framework for cognitive processes.

## *Results*

It was previously suggested[1] that the maximal cognitive stability $S_{max}$, would be given by an expression for the maximal interaction of non-identical elements such that:

$$S_{max} = K \ln \frac{n!}{k_1!k_2!..k_k!..k_m!} \quad (1)$$

where K is a constant, n! the number of permutations between n elements reflecting the total number of interactions between them, and $k_1!$, $k_2!...k_m!$ the contribution to n! given by the interactions between like elements $k_k$. In this expression, every element of type k interacts directly or indirectly with all other elements. The value of $S_{max}$, or the 'maximal cognitive stability' increases when the interactions between different elements are favored at the expense of interactions between identical elements, as, for example, in the case of biological 'symbiosis'. $S_{max}$ has no informatory content and its magnitude for any elemental combination changes as a result of the quantity of interactions rather than the type of elements. If, for example, the expression in eq. (1) is weighted by one more occurrence or by the addition of an element that is already present in one or more copies, the values of $S_{max}$ will increase, but the combination of interactions between the elements will not change qualitatively. If this is to happen, a different element has to be added or all copies of another one subtracted from the cluster of interacting elements in eq. (1).

There is nothing in this expression which excludes interactions between like elements unless they are deliberately minimized, in which case k equals 1. Applying this exclusion principle to eq. (1) leaves the requirement that there be at least and at most one sub-element of the same kind to provide the one permutation in the denominator which is needed for the expression to be defined. Furthermore, if $k = 1$ for all elements in eq. (1), and the composite element itself is included with other sub-elements in the elemental cluster, then a minimum requirement to obtain an expression of finite stability is that the composite element interacts with one sub-element. Then the cognitive stability will be proportional to ln2, but will increase with each additional sub-element which is defined. The composite element itself is the sub-element which is responsible for the communication with other composite elements in most instances, and therefore the situation is somewhat reminiscent of the case of the valence electrons of atoms which mediate the interactions in composite molecules while other electrons are comparatively inert.

Some of these ideas can be illustrated using line elements (fig. 1). First of all, the stability of a concept is devoid of specificity and all its informatory content lies in the type of interaction between its sub-elements (complementarity). A straight line is identical with any other straight line, since in it there are no interactions between line elements of different spatial orientations or positions. However, as soon as there are orientational or positional interactions, the line elements differentiate to acquire specific meanings related to their interactions with other elements. The stability of a concept can then be illustrated by its size, when a big composite element (in the figure, a square) is more resistant than a small one towards changing the orientation of one line element. Also, repetitive occurrences of a composite element, i.e. repetitive interactions between its sub-elements, increase its stability and resistance towards changes (fig. 1).

The implication of the expression in eq. (1) is therefore that each sub-element of a composite element interacts directly, or indirectly (via an observer), with all other elements to provide stability. Superfluous interactions between like elements are then excluded from the attention and interactions between different sub-elements reinforced by repetitive occurrences of the composite element, which maximizes the cognitive stability. That the 'whole picture', or context, is important for the meaning of one piece of information is well appreciated in cognitive science, and can be illustrated (if one is cautions about the fact that naming an element is not equivalent to the element itself) using line elements that form letters and words (fig. 1E) or other patterns. That a context in which an element occurs determines its meaning also implies that non-causal spatial and temporal neighborhoods of elements govern the state of evolution of the cognitive process.

Each element of type k can interact with $C_k$ other elements, where $C_k$ is the 'complementarity' of the element.

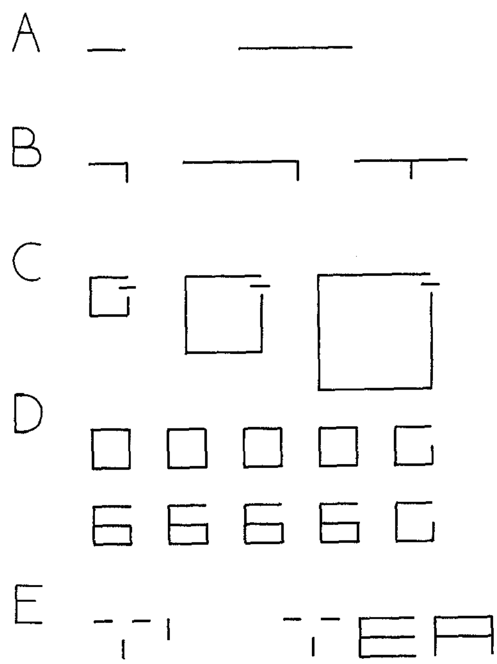


Figure 1. Illustration of cognitive properties of lines. In A, the identity of long and short straight lines is illustrated. In B, the interaction between two line elements characterized by different orientations is illustrated. The first of the three composite elements in B is different from the second one, where the horizontal line is longer and thereby provides a particular positional interaction with the vertical line. In the third composite element, the type of interaction between the horizontal and vertical lines is different from that in the two previous elements. In C, a composite element, a square, is formed by either four short or four long line elements. The big square provides more cognitive stability and is resistant towards changing the orientation and, thus, the identity of one line element. In D, the increased cognitive stability is provided by repetitive occurrences of either a square or a schematic representation of the number '6'. Depending on whether the upper or lower horizontal lines with these representations is read, the final assembly of lines is interpreted as being either an incomplete square or an incomplete number '6', although objectively, it is another representation. This shows that the stability of a concept is reinforced by repetitive occurrences. In E, the importance of the context for the meaning of line elements is illustrated. The first set of lines is not interpretable, whereas the second set of lines is interpreted as the word 'tea'. This shows that elements interact beyond their nearest neighbors irrespective of sequence, which is the conceptual basis of the expression in eq. (1).

The number $C_k$ is given by:

$$C_k = n - k_k \qquad (2)$$

Where n is the total number of interacting elements and $k_k$ the number of elements of type k. A cognitive process is characterized by its 'complexity', Q:

$$Q = m - 1 \qquad (3)$$

which number equals the number of interaction surfaces of any element, where m is the number of species of different elements. Both $C_k$ and Q are quantal numbers; an element is either present or absent, and an interaction is either complementary or impossible. The relation between $C_k$ and Q is such that:

$$C_k \geqslant Q \qquad (4)$$

In other words, the number of species of elements cannot exceed the total number of elements and when an elemental interaction has been defined, the contained elements have also been defined.

It is convenient to use the term 'cognitive plasma' to denote a composite element of the type that has been described. Then, a normalized cognitive plasma is one in which the equality in eq. (4) is valid. In a weighted cognitive plasma, the inequality $C_K > Q$ holds, and there is more than one interaction between any non-identical elements. A weighted cognitive plasma is illustrated in figure 2.

In a weighted cognitive plasma, the interactions between its sub-elements are not evenly distributed, and each element is characterized by a 'flux density' of interaction, $\varrho_k$, given by:

$$\varrho_k = \frac{C_k}{Q} \qquad (5)$$

$\varrho_k$ reflects the uniqueness of an element and its importance for the maintenance of the composite element of which it forms a part. This parameter emerges only by the specific interdependence of the elements within the complex system and changes considerably if the same elements are part of a different system. Examples of elements with a high value of $\varrho_k$ would be, inter alia, negations in sentences and exact mathematical expressions that describe a variety of empirical phenomena. In a living cell, a segment of DNA coding for an important enzymic pathway, or in a society, any idea of considerable public exposure would fit to a high value of $\varrho_k$. In any composite element which has formed from sub-elements, the composite element itself is part of the elemental cluster and has the highest value of $\varrho_k$. The reason why $\varrho_k$ is called 'flux density' is clear from the following. When $Q = m - 1$, then all elements are complementary and the cognitive process is independent of time. However, if there are fewer interactions than permutations of non-identical elements, then the 'reduced complexity', q, can be defined to replace Q, in such a way that:

$$q < m - 1 \qquad (6)$$

This means that a certain time has to pass before all the interactions that define a cognitive plasma can take place, that the interactions take place one at a time, and thus that there is a 'flux' of interactions. Apparently, the reduced complexity is related to time and it is appropriate to relate q specifically to the time t and the segment of time when q is defined, $\Delta t$, in such a way that:

$$\frac{Q}{q} \propto \frac{t}{\Delta t} \qquad (7)$$

Equation (7) relates the complexity of a cognitive process to time and implies that for a process that is repeatedly observed, either the amount of elements and possibilities of interaction (Q) increases, or the number of observable interactions (q) decreases. This can be interpreted to mean that observing a cognitive process (q and $\Delta t$ are defined) deprives it of the means of continuing, or in other words, its interactions are extracted from the process. That can only be counterbalanced if more elements evolve and Q increases. In the expression in eq. (7), a segment of time of observation is only defined if it contains at least one interaction.

Equation (7) is intuitively appealing since the number of elements and concepts that can be conceived by a human being increases as a function of time, the amount of experiences, and education. It is common in mental cognitive processes and probably related to the equation that during periods diversified activity (that is, increasing values of Q) the length of time is not experienced in the same manner as if it is filled with routine preoccupations (constant value of Q). The equation in (7) has previously been applied in the empirical definition of 'Intelligence Quotient' (IQ), where the intellectual achievement is divided by the actual age.

There is a certain probability, $p_{kl}$, that a certain element k will interact with a certain non-identical element l. For a normalized cognitive plasma, this probability is $p_{kl} = 1/C_k$. For a weighted cognitive plasma, the probability is:

$$p_{kl} = \frac{k_l}{C_k} \qquad (8)$$

where $k_l$ is the number of elements of type l with which the element k can interact.

For a cognitive plasma of reduced complexity as compared with one of full complexity, a flux density of interaction for each sub-element can only be obtained if longer time periods are considered. There are Q probabilities of the type in eq. (8) and in a system of reduced complexity, q of these are attended to at any given interval of time. This means that by introducing reduced complexity, competition is introduced among the elements of the cognitive plasma. The competition, or 'attention', can be interpreted in such a way that the interactions between the elements are mediated by even smaller sub-elements (fundamental elements, cf. ref. 1) which have a finite number (q) and hold the composite element together by spreading over its whole structure, shuttling between its sub-elements[1]. Then, a contextual element can be either 'populated' or not 'populated' with fundamental elements, or means of interaction.

So far, the equations and definitions have described processes that do not change qualitatively with time, at least if averages over sufficiently long time periods are considered. Time evolution of the cognitive process can be introduced by iterative reinforcement of the probability in eq. (8) each time the interaction occurs, and by assigning a relaxation time described by $e^{-A\tau}$, to the probability that a subsequent interaction of the same type will occur again, starting from the first of the interacting elements. This is justified by the occurrence of exponential curves both for learning and retention in human experimental psychology.

By allowing a cognitive process of reduced complexity to evolve, that is, to change qualitatively, certain interactions will be favored at the expense of others, and it is appropriate to denote such a process as a 'cognitive string'. If one element l is added with each interaction from k to l, the probability of such interactions according to eq. (8) will ultimately approach unity. This will occur comparatively rapidly in a stable environment, while in an open complex system in a changing environment the process is obliged to evolve in order to match its interactions with the external elements and absolutely certain interactions are less likely to appear. In a process of reduced complexity, the probabilities of interaction between k and other elements than l will, if they are sufficiently small originally, approach zero. This will also contribute to that the quotient in eq. (8) approaches unity.

The flux condition discussed above means that any element which is populated must become depopulated in a process of full complexity whereby iterative reinforcement means that $k_k = C_k$. Since $C_k = n - k_k$, it follows that $n = 2k_k$ and, if the interaction between k and l is certain, that $k_k = k_l$. Therefore, any composite element can be defined by naming two sub-elements, of which one may be related to its context and the other one may be 'internal'. This may be interpreted to mean that each element corresponds to an element which is its mirror image, or name, that there is directionality of interaction, i.e. that the input and output of the interaction are different and located in a way that permits interaction between mirror images, and/or that the elements are characterized by two types of 'spin' and otherwise identical. In terms of the expression in eq. (1), the most obvious interpretation is that even a single element must form a stable concept ($S_{max} = K \ln 2$). From a psychological point of view, it is doubtful if a 'mental' element exists until it has been reexperienced, as for example in scientific practice, where

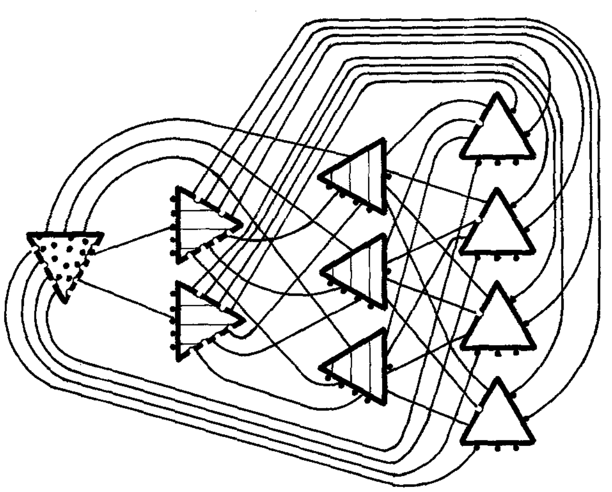

Figure 2. Illustration of a weighted cognitive plasma. Elements are illustrated by triangles of different patterns, complementarity by dots and indentations and interactions by lines connecting complementary surfaces.

it is customary to reconfirm the result of an experiment at least once before it is reported. Therefore, to define a normalized cognitive plasma in an evolving string by naming two sub-elements seems to be amply justified.
It is obvious that a probability of close to unity of an interaction between two elements is equivalent to saying that a new composite element has formed from the two original elements, a conclusion which is reinforced by the finding that such an element fits into the definition in eq. (1). Therefore, it is reasonable to include iterations in the definition of the probability of interaction between two elements in an evolving cognitive string. Iterations have also been used previously in the mathematics of cognition.
Considering the above, the appropriate definition of the probability, $p_{kl+}$, that an interaction between two elements k and l in an evolving cognitive string will occur one more time after it has once occurred, and provided that the first element, k, is populated, i.e. that a reduced complexity does not deprive the two elements of the means of interaction, is:

$$p_{kl+} = \frac{k_l + 1}{C_k + 1} e^{-A_{kl}\tau} \qquad (9)$$

In this expression, A is the 'association parameter'. This parameter is a function of systemic influences on the probability of interaction between two elements and can

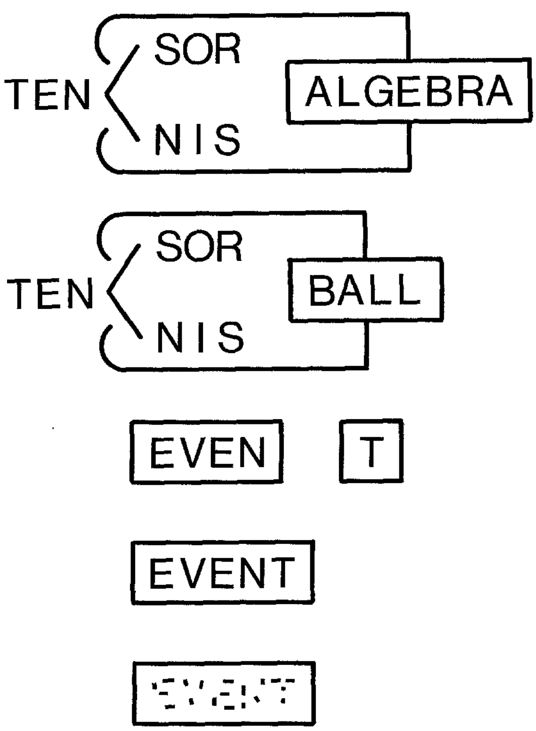


Figure 3. Illustration of the importance of the context (association) on the probability of interaction between sub-elements. The context is represented by words and sub-elements by letters or lines. The illustration in A shows that the probability of interaction between the letter 'N' in 'TEN' and either 'S' or 'N' is affected by the context, namely the word element which follows last. Due to previous occurrences in the mind of the reader either 'SOR' or 'NIS' is selected to be the appropriate continuation from 'TEN', depending on the context. In B, 'elementary reductionism' is further illustrated, i.e. the fact that the sub-elements interact beyond their neighbors. This is illustrated on the level of letter elements and on the level of line elements.

sustain or quench a memory, $p_{kl+}$, but never reinforce it. Also, more than two elements may join to form a new element. This may happen, for example, if an interaction subsequent to another one is reinforced through the association parameter, a situation that will be further analyzed below.

The relaxation time, $\tau$, is relevant only to a particular element or elemental interaction. It is zero at the time of the interaction and then determines the decay of the probability of another interaction of the same kind between populated elements according to the exponential term in eq. (9). When the same interaction again occurs, $\tau$ again becomes zero and the decay of the probability of a subsequent interaction starts from the beginning. For each repetitive interaction, the quotient $k_l/C_k$ and the association parameter, A, will have changed slightly. $\tau$ should be distinguished from the time of evolution of the cognitive process, t, which is the same at any moment for all elemental combinations.
Thus, each interaction (and each composite element, see below) is characterized by two different times, its own time, $\tau$, and a universal time, t, which is the same for all elements. In addition, intervals of time, $\triangle\tau$, and $\triangle t$ can be defined, which are the time lapse between subsequent examinations of the process and the duration of the observation, respectively.
Similar equations to that in eq. (9) hold for any other elements i, j, m, ... with which k is capable of interacting. Since the sum of all target elements $k_i$, $k_j$, $k_l$, ...$k_m$ with which an element k can interact is equal to $C_k$, the sum of the corresponding probabilities is equal to 1 for time 0. If $\tau > 0$ and $A \neq 0$ for any elemental interaction, then the sum of these probabilities $\sum_{l}^{m} p_{k+}$ will be less than unity and there will be a certain probability that the interaction does not take place and that the composite element goes out of the context:

$$\sum_{l}^{m} p_{k+} \leqslant 1 \qquad (10)$$

The expression in eq. (10) ascertains that even probabilities of less than 0.5 can be reinforced by iteration. Certain probabilities characterized by a high value of the association parameter will soon vanish for any real time periods of $\tau$ (cf. eq. 9). This means that certain elements will be out of consideration for interaction from a certain element k and that a 'reduced complementarity' $c_k$ will emerge to replace the 'full' complementarity' $C_k$, such that:

$$c_k < C_k \qquad (11)$$

The definition of $c_k$ is arbitrary and depends among other things on A and $\tau$. The relationship is such that the more common an interaction has been as reflected by the association parameter A, the lower should be the value of $c_k$. The appearance of the reduced complementarity by the exclusion of certain elements from the possibility of interaction from k, during the course of relaxation from a previous interaction from k to l, will efficiently speed up the formation of new concepts by allowing the corresponding quotient $k_l/c_k$ to approach unity more rapidly than would be the case with full complementarity and the quotient $k_l/C_k$.

It is clear from the equations (9–11) that for short time periods of $\tau$ and $\Delta\tau$, the memory of the micro-interactions between neighbors will determine the behavior of the process. If, however, the cognitive process is allowed to 'forget' by increasing $\Delta\tau$, the time period between subsequent observations of the process, then the context, reflected by the association parameter, A, will become increasingly important. The latter means that the narrower the time horizon for deterministic forecasting (where the system still behaves like a 'machine') the wider the time horizon not allowing non-deterministic forecasting but only probabilities governing the behavior of the system towards disturbance and subesequent evolution within its context. Therefore, there may be a strict mathematical relationship between e.g. the implementation of an internal constraint (such as an evolutionary strategy speeding up decision-making) within a system, and the clashes of such a system when adapting to the external constraint of a changing and diversified environment.

In mental cognitive processes, the existence of the relaxation time would be responsible for the well-known experience that a problem can be easier to solve after it has not been dealt with for some time. After the relaxation time has passed, it may be easier to 'put the pieces together' in a new combination which reflects increased cognitive stability. This may also explain the fundamental difference in the cognitive effects of learning by making a mental note or by understanding the connections. Even in molecular cognitive processes the exponential time parameter is relevant, for example when plasmids are inserted into host DNA or when breaks of DNA are connected to new segments. This is an example of the ever-changing 'dynamic' nature of cognitive processes. The relaxation time would also be relevant to numerous processes in human society – anything from the rewriting of history from present day perspectives, to the impossibility of keeping objective computer files about citizens.

One arbitrary way of defining $c_k$ is to demand that the sum of the probabilities of interaction in eq. (10) approach unity. This could be done, for example, by amplifying the probabilities obtained after a time $\tau$ by a factor $e^{-AK}/\Sigma p_{k+}$. In this expression, $\Sigma p_{k+}$ is the sum of the probabilities of elemental interactions in eq. (10) and $e^{-AK}$ is a weighting factor which again contains the association parameter, A. The use of the association parameter here is justified by the fact that some elemental interactions occur in some contexts but not in others. Namely, in some contexts, the probabilities of those interactions are amplified until they reach certainty, which requires in most cases that the full complementarity $C_k$ is replaced by a reduced complementarity, $c_k$. One way of defining the amplified contextual probability of elemental interactions from the element k to the element l is therefore:

$$P_{kl+} = \frac{k_l + 1}{c_k + 1} e^{-A_{kl}(\tau + K)} \frac{1}{\Sigma p_{k+}} \qquad (12)$$

where $P_{kl+}$ is an amplified probability and $c_k$ is a reduced complementarity anywhere in the interval $k_l \leqslant c_k < C_k$. In this expression, the importance of the relaxation time, $\tau$, has been overruled by that of the previous contexts in which the interactions have occurred as reflected by the feature that when $\tau$ is 0, then the probability of interaction is determined primarily by $k_l$, $c_k$ and A.

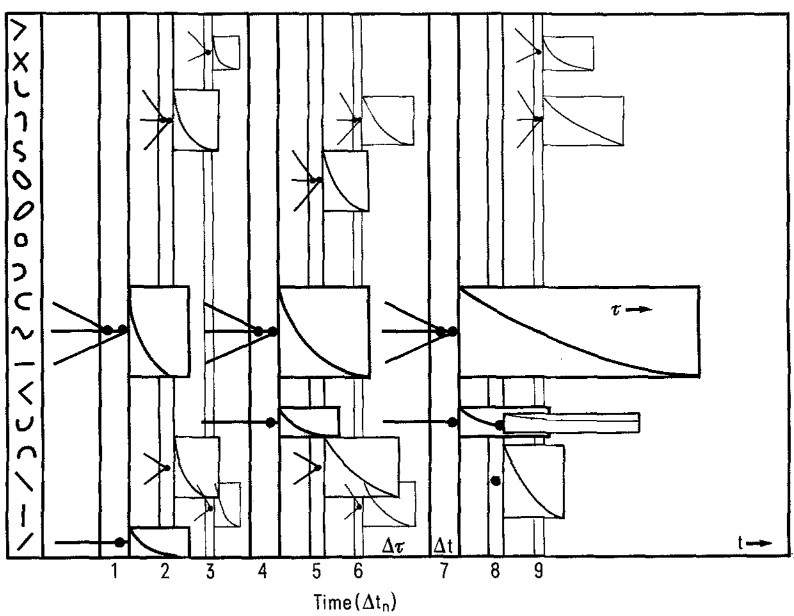


Figure 4. Illustration of the evolution of a cognitive process. At time zero only sub-elements exist and as time passes more and more complex elements are formed. Interactions are illustrated by straight lines connecting y-values representing various elements. Each interaction and each occurrence of a composite element is followed by a decay of the probability of a subsequent interaction of the same kind. The time of observation is illustrated by a segment of time $\Delta t$, and the time between subsequent observations illustrated by $\Delta\tau$, on the x-axis. Interactions are illustrated as dots on the lines which precede a decay of a subsequent probability. In the figure, two parallel cognitive processes are running. In the time intervals 1, 4, and 7, in the middle of the figure, an interaction is gradually enforced by repetitive occurrences, which leads to a slower decay of the probability of a subsequent interaction. In the upper interaction in time segment 8, an element is being populated a second time before the probability of interaction has decayed to a minimal value and within the time lapse between two observations of the process, $\Delta\tau$. The lower interaction in time segment 8 represents that a new element has formed from three separate elements which previously were populated in the segments 2, 3 and 5, 6. The decay curves have not been computed exactly and only represent qualitative differences.

A cognitive string which obeys eq. (12) will always remain populated, but at the expense of its freedom to evolve in an exactly weighted direction. Therefore, a cognitive process will mature according to eq. (9) but may at any moment decide to oversimplify things and obey eq. (12) or any similarly constructed expression. The inequality in eq. (10) can be illustrated by the freedom of thought to evolve in any suitable direction or to leave its subject until sufficient evidence has accumulated to make a proper choice. On the other hand, if the interaction probabilities have been artificially amplified proportionally to their previous occurrence within a certain context, then the situation more resembles a dream where improper choices are possible but sometimes evoke a definable context or interpretation. If the cognitive string has been partly reinforced on a few or many previous occasions, as in the case of human language, then populating it continuously by putting $\Sigma P_{k+}$ equal to 1 may produce anything from 'Finnegans Wake' to rigorous textbooks of law.

These lines of thought may be qualitatively illustrated by the probability of interaction between letter elements, which varies according to the context (fig. 3). In certain contexts, certain letter strings are natural while in other contexts, the same strings are out of the question, due to the lack of previous occurrences or 'interactions' between word and letter elements of which they form a part. This means that the higher elements (words) interfere with the letter elements and their interactions and also that the letter elements interact beyond their nearest neighbors. This may be called 'elementary reductionism'[1], an essential feature of cognitive processes, which is illustrated in fig. 3. The previous description of cognitive processes can be illustrated as in figure 4, where many of the necessary concepts, namely elements, composite elements, interactions, t, $\tau$, $\triangle$t, and $\triangle\tau$ are illustrated.

The interactions require that the elements are spatially or temporally close. That spatially close elements are contextual is obvious. The spoken language can be taken as an example that temporal proximity is equivalent in this respect. In the spoken language, word elements that occur within a time interval of $+/-\triangle t/2$ counted from any moment, t, are understood to be related. The same holds for events, like e.g. human actions or traces from nuclear particles in bubble chambers. If the same elements or events occur in the same combination many times, then the situation can be regarded as if the elements are populated many times and interact many times, which increases the cognitive stability according to eq. (1).

Previously, it was mentioned that contextual elements can become depopulated and that competition, or attention, among contexts and elements is introduced by replacing the full complexity Q with the reduced complexity q. To regard elements as being populated or not populated with fundamental elements of a defined quantity q, lends to cognitive processes a similarity to economics. Namely, the quantity q corresponds to the money or any other symbolic entity which mediates economic interactions between elements on the economic market. If an element is depopulated in terms of the present formalism, this corresponds either to buying something or depositing the money in a bank. If the deposit is not recirculated, then there will be increased competition for the available money and only the most necessary things will become affordable. This corresponds to a situation when only a few elements are populated and the value of q is low. On the other hand, if the value of q is high, more elements are populated at the same time. This, of course, is a more general way of saying that more things can be afforded and bought for money. The latter situation will speed up the evolution of the 'economic' cognitive process since more combinations can be tried at the same time.

The reduced complexity, q, is not a fixed quantity. Therefore, when a context is depopulated, this does not automatically imply that another one becomes populated. The condition for a populated element to cause an interaction is given by eq. (10). A similar condition can be formulated for an element to become populated even if it is not part of the context originally. Namely, when a certain composite element has been depopulated, any element which has previously occurred within a critical time $+/-\triangle t/2$ framing the moment when the first element was being depopulated on previous occasions will have a non-zero probability of interaction with that element. Therefore, any elements which have occurred approximately at the same time as the composite element on previous occasions will earn probability according to the mechanism described in eq. (9) and will with each iteration become more and more likely candidates to take up the interacting fundamental element. This follows from the equivalence of spatial and temporal neighborhoods as determinants for the evolution of the cognitive process and leads to the cognitive string gradually developing into a weighted cognitive plasma.

Thus, after a contextual occurrence of a new element m, the probability of a subsequent occurrence is given by an equation identical to eq. (9), only with l replaced by m. The probability that a new element will be added to the context may also by choice be given by eq. (12) or any other amplified probability or reduced complementarity. That two contexts which originally are not related can become related implies that at least two non-interacting cognitive strings may operate independently at the same time. If the same elements happen to occur within a time $\triangle$t more than once, there is an increasing probability that they will co-occur again.

Previously, it was mentioned that q is not necessarily a fixed quantity and that the velocity of a cognitive process $q/\triangle t$ may vary. However, there may exist a certain preferred velocity, or mean velocity towards which the process is always adjusted, whatever the specificity of the elemental interactions may be. Then, the observed velocities may follow a normal distribution around this mean velocity and large variations from it may be unlikely. This situation has been implemented in the neural synapse, where, due to metabolism, transmitter substance is continuously accumulated in the synaptosomes and must be released before it causes the nerve endings to swell. In this case, the mean velocity of the cognitive process may be proportional to the mean frequency of synaptosomal release of transmitter substance.

That the velocity of a cognitive process is finite means that when a new element $m_c$ is added to a cluster of elements, another element $m_d$ may be subtracted or dissociated from the cluster. This leads to a criterion for the stabilization of a cognitive process (cf. ref. 1) according to which:

$$\frac{(k_c + m_c)!}{k_c!\, m_c!} > \frac{(k_d + m_d)!}{k_d!\, m_d!} \tag{13}$$

where the index c represents elements which belong to or are added to the cluster, and the index d represents elements that are involved in dissociation of an element m. This expression implies that the stability of a cognitive process of full complexity increases if there are more copies of an added interacting element than of a subtracted element. In other words, common interacting elements are favored at the expense of less common interacting elements if there is competition between them. During this process, the composite element is qualitatively changed.

As for the association parameter, it is obvious that it should somehow reflect the composition and stability of a weighted cognitive plasma as given in eq. (1) in four-dimensional space, where time is one dimension. The contained elements are defined by the sub-elements having the highest flux densities, $\varrho$, and, for each element k, in addition at least one or an arbitrary number of other sub-elements. For each element k and each value of $\varrho_k$, there is a group of other elements with lower values of $\varrho$ which are contained in k and which can be called upon and included in the expression for cognitive stability at any time. This increases the associative power of the expression of cognitive stability in eq. (1) more than the mere repetition of the same higher element k. Therefore, just mentioning an abstraction does not have as much informative value as if it can be exemplified at the same time. On the other hand, it is obvious that elements with a sufficiently high level of abstraction must be chosen in order to exclude information that is not relevant. By doing so, it is possible to define any element precisely by measuring previous occurrences of a few sub-elements which are comparatively specific (in terms of a high value of $\varrho$) and the contexts in which the element has occurred. While the sub-elements can be observed in the spatial neighborhood of the element, the contextual elements can be observed in the temporal neighborhood $\Delta t$, as defined in eq. (7). $\Delta t$ is related to the reduced complexity q, which means that human attention can be concentrated only on a few elements at a time and irrelevant information is forgotten. This is particularly evident in the 'anatomic learning phase' of higher organisms like man during the first months after birth, and the imprinting of a codified pattern of the environment. Only on the basis of such a 'hardware' – ground pattern can further perceptions be handled without danger of confusion[2]. Subsequently, the accumulated experience becomes increasingly important since it adds weight to the occurrence of individual elements and contexts in the actual situation.

The probability of a certain interaction from one element k to another element l is consequently given by the expressions eq. (9) and eq. (12) in which the association parameter A is the inverse of the maximal cognitive stability of the elements which have occurred within a time $\Delta t$ framing the previous interactions between k and l:

$$A = \frac{1}{S_{max}} \tag{14}$$

Some of these elements have already occurred in the particular context at time t, while others may frame the interaction in the immediate future. Therefore, the association parameter changes continuously as the specificity of the context is increased. With only a few elements specified the decay of the probability of an interaction between k and l at a given rate of the cognitive process $q/\Delta t$, may be so fast that the interaction does not occur. However, as other contexts are specified or anticipated by interactions involving other fundamental elements, the association parameter may become small enough to make probable an interaction between k and l.

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# Short Communications

## Responses of monkey, rabbit and dog internal carotid arteries to atrial natriuretic factor

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*Summary.* Effects of atrial natriuretic factor (ANF) on monkey, rabbit and dog internal carotid arteries were investigated. ANF caused a concentration-dependent relaxation in arterial strips submaximally precontracted with noradrenaline, 5-hydroxytryptamine, or high-potassium solution (10–30 mM). The response was greatest in the monkey arteries and least in the dog arteries. These results suggest that there is a marked species difference in the ANF-induced relaxation of the internal carotid arteries.
*Key words.* Atrial natriuretic factor (ANF); internal carotid artery; vasodilation.

Mammalian atria contain a potent natriuretic substance[1,2], which could be a novel peptide hormone of considerable importance for renal and cardiovascular homeostasis. This atrial natriuretic factor (ANF) also causes relaxation of isolated vascular and nonvascular smooth muscle preparations[3,4]. No detailed reports, however, have been made of the effects of ANF on cerebral arteries, except that by Faison et al.[5], who demonstrated a slight ANF-induced vasodilation in rabbit basilar artery. The